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DEVELOPMENT AND ANALYSIS OF A TURBULENCE MODEL FOR BUOYANT FLOWS

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ABSTRACT

A Turbulence Model for Buoyant Flows (TMBF) has been developed. TMBF is a combination of a first-order 2-equation model for the turbulent transport of momentum and of a second-order 5-equation model for the turbulent transport of heat. TMBF represents a compromise between the classical k-ε-σ_t model and the Reynolds stress model, but it is clearly an improvement of the k-ε-σ_t model for turbulent flows where the turbulent transport of heat is complex, such as buoyant flows, and the Reynolds analogy is not valid. In order to extend the range of TMBF to low Peclet numbers, a new model relationship has been introduced. TMBF has been implemented in the FLUTAN Computer Code and it has been validated by means of experimental data from turbulent flows in forced and mixed convection. The numerical results obtained by using TMBF show that stratified flows and buoyant effects in mixed convection are well predicted. They cannot be adequately calculated by means of the k-ε-σ_t model.

1. INTRODUCTION

The statistically-averaged conservation equations contain unknown correlations, the turbulent stresses and heat fluxes, which represent the turbulent transport of momentum and heat. In order to close the set of conservation equations, these correlations have to be replaced by turbulence models.

A well-known class of turbulence models is based on the eddy viscosity / heat diffusivity concept. The eddy viscosity ν_t and eddy heat diffusivity Γ_t are respectively introduced in the terms representing the turbulent transport of momentum and heat by a mean gradient approach. The eddy heat diffusivity is assumed as isotropic and is linked to the eddy viscosity by a fixed turbulent Prandtl number σ_t. This implies that the turbulent transport of heat is assumed to be strictly analogous to the turbulent momentum transport. It is called Reynolds analogy. These assumptions work well for a wide class of forced convective flows. They are not valid for buoyant flows where the turbulent transport of heat can be strongly anisotropic and not analogous to the transport of momentum. In contrast to this, a second-order description of the turbulent transport of heat, which means the use of transport equations for the turbulent heat fluxes, is not constrained by these assumptions. Therefore, in order to develop a turbulence model for buoyant flows it is reasonable to use a second-order model for the turbulent transport of heat.

Further development and analysis of a Turbulence Model for Buoyant Flows (TMBF), whose old version was already presented in a previous work (Carteciano 1995), are presented in this paper. TMBF is a combination of a first-order 2-equation model for the turbulent transport of momentum and of a second-order 5-equation model for the turbulent transport of heat. The turbulent stresses are calculated assuming an isotropic eddy viscosity and solving the transport equations for turbulent kinetic energy k and for its dissipation rate ε. The three turbulent heat fluxes $\overline{U_i'T'}$ are determined by means of transport equations. Moreover, transport equations for the variance of temperature $\overline{T'^2}$ and its dissipation rate $\varepsilon_{T'}$ are used in the description of the turbulent transport of heat. In order to extend the range of the transport equations for $\overline{U_i'T'}$ to low Peclet numbers, a new model relationship has been developed. TMBF does not introduce six additional transport equations for the turbulent stresses and thus differs from the so-called Reynolds stress model. However, the calculated turbulent stresses and heat fluxes are no longer related through a fixed turbulent Prandtl number σ_t.

TMBF has been implemented in the FLUTAN Computer Code, which is a highly vectorized computer code for 3-D, single phase, thermo- and fluid-dynamic problems in complex geometries (Willerding and Baumann 1996).

The performance and accuracy of flow simulations using TMBF has been tested for the following kinds of flows: a) 2-D forced convection flows, for which the assumptions of isotropy of eddy heat diffusivity and of Reynolds analogy are not valid; b) 2-D buoyant mixed convection flows, for which the turbulent transport of momentum is close to isotropic.

In the model analysis the following aspects are considered: the description of the turbulent transport of heat using a second-order model in combination with the k-ε model; the new model relationship in the transport equations for $\overline{U_i'T'}$ and the description of buoyancy.

The results are compared with experimental data in parallel with calculations using the standard k-ε-σ_t model, in order to show how TMBF improves the k-ε-σ_t model.

2. TMBF

The turbulent shear stresses are modelled in TMBF using the gradient assumption of Boussinesq:

$$-\overline{U_i'U_j'} = \nu_t\left(\frac{\partial \overline{U_i'}}{\partial x_j}+\frac{\partial \overline{U_j'}}{\partial x_i}\right)-\frac{2}{3}k\delta_{ij}. \qquad (1)$$

The eddy viscosity ν_t is introduced by this assumption. The distribution of ν_t , which is assumed to be isotropic, is calculated using the Kolmogorov relationship:

$$\nu_t = c_\mu f\left(\frac{P_k+G_k}{\varepsilon}\right)f_\mu\frac{k^2}{\varepsilon}. \qquad (2)$$

This relationship contains an empirical coefficient c_μ with its correction function $f\left(\left(P_k+G_k\right)/\varepsilon\right)$ from Rodi (1972) and Carteciano (1996) and the damping function f_μ, which is necessary to extend the validity of this relationship to low Reynolds numbers. In this model, the formulation for f_μ which was proposed by Nagano and Kim (1988) is used:

$$f_\mu = \left[1-\exp\left(-\mathrm{Re}_\tau/26.5\right)\right]^2. \qquad (3)$$

In order to calculate the eddy viscosity, the transport equations of the turbulent kinetic energy k and its dissipation rate ε are solved:

$$\frac{\partial k}{\partial t}+\overline{U_i}\frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}+\nu\right)\frac{\partial k}{\partial x_i}+P_k+G_k-$$

$$\varepsilon-2\nu\left(\frac{\partial\sqrt{k}}{\partial x_i}\right)^2, \qquad (4)$$

$$\frac{\partial \varepsilon}{\partial t}+\overline{U_i}\frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}+\nu\right)\frac{\partial \varepsilon}{\partial x_i}+$$

$$\frac{\varepsilon^2}{k}\left[c_{\varepsilon 1}\frac{P_k+G_k}{\varepsilon}\left(1+c_{\varepsilon 3}Ri_f\right)-f_2 c_{\varepsilon 2}\right]+$$

$$\nu\nu_t\left(1-f_\mu\right)\left(\frac{\partial^2\overline{U_i}}{\partial x_j\partial x_l}\right)^2, \qquad (5)$$

$$P_k = \nu_t\left(\frac{\partial\overline{U_i}}{\partial x_j}+\frac{\partial\overline{U_j}}{\partial x_i}\right)\frac{\partial\overline{U_i}}{\partial x_j}, \qquad (6)$$

$$G_k = -\beta g_i\overline{U_i'T'}, \qquad (7)$$

$$f_2 = 1-0.3\exp\left(-\mathrm{Re}_t^{\,2}\right). \qquad (8)$$

The turbulent diffusive terms are modelled with mean gradient assumptions (Rodi 1972). For the production and sink terms in the transport equation for ε, the modelling of Jones and Launder (1972) is used with a correction function f_2 for the empirical coefficient $c_{\varepsilon 2}$ in order to extend the validity of the standard value of $c_{\varepsilon 2}$ to low Reynolds numbers. To consider the buoyancy influence on ε, this modelling is modified introducing the buoyancy term G_k through a correction term which contains the flux Richardson number Ri_f .This number is defined as Ri_f =-0.5 $G_{V'}$ / (P_k + G_k) where $G_{V'}$ is the buoyancy production of only the lateral energy component V' (Rodi 1980).

Both equations contain the buoyancy term G_k, which depends on the turbulent heat fluxes. This is an important term for a turbulence model for buoyant flows because it represents the only mechanism in which the temperature field affects the momentum field by means of turbulent buoyancy. TMBF incorporates a detailed modelling of G_k in that the transport equations for the turbulent heat fluxes are solved:

$$\frac{\partial\overline{U_i'T'}}{\partial t}+\overline{U_j'}\frac{\partial\overline{U_i'T'}}{\partial x_j} = \frac{\partial}{\partial x_j}\left[c_{TD}\frac{k^2}{\varepsilon}+\frac{\Gamma+\nu}{2}\right]\frac{\partial\overline{U_i'T'}}{\partial x_j}-$$

$$\left(\overline{U_i'U_j'}\frac{\partial\overline{T}}{\partial x_j}+\overline{U_j'T'}\frac{\partial\overline{U_i'}}{\partial x_j}\right)-G_{U_i'T'}+\pi_i+Z_i. \qquad (9)$$

The molecular and turbulent diffusive terms are modelled by mean gradient assumptions (von Weissenfluh 1984 and Launder 1978). The modelling of the pressure-scalar gradient correlation π_i of Monin (1965) and Launder (1975) contains also a wall function in order to consider the damping effect of the wall on the turbulent heat flux perpendicular to the wall (Gibson and Launder 1978):

$$\pi_i = -c_{T1}\frac{\varepsilon}{k}\overline{U_i'T'}+c_{T2}\overline{U_j'T'}\frac{\partial\overline{U_i}}{\partial x_j}+$$

$$c_{T3}\beta g_i\overline{T'^2}-c_{T4}\frac{\varepsilon}{k}\overline{U_{[n]}'T'}\delta_{i[n]}\frac{k^{3/2}}{x_{[n]}\varepsilon}. \qquad (10)$$

There is no summation over the index n, which indicates the normal direction to a wall.

A new modelling term Z_i is developed and introduced in the transport equation (9) in order to cover the range of low Peclet numbers:

$$Z_i = -\frac{1+\Pr}{2\sqrt{\Pr}\sqrt{R}}\left(\frac{\varepsilon}{k}\right)\exp\left[-c_{T5}\left(\mathrm{Re}_t + Pe_t\right)\right]\overline{U_i'T'}. \qquad (11)$$

This modelling bases on ideas of Shikazono and Kasagi (1990) modified by applying it to a combination of terms of the turbulent heat flux equations, for which it was originally not intended. In fact, the term Z_i takes into account the dissipation of the heat fluxes, which becomes important at low Peclet numbers, and the modification of the modelling for π_t which is necessary to extend its validity to low Peclet numbers. Therefore, the contribution of Z_i in the transport equation (9) must be negligible at high Peclet numbers but must become important at low Peclet numbers. For this reason, an exponential function of the sum of turbulent Reynolds and Peclet numbers, supported by the DNS investigation of Wörner and Grötzbach (1995), is suggested here. The complete expression of this modelling can be derived by dimensional analysis (Carteciano 1996).

The transport equation (9) contains a buoyancy term $G_{U'T'}$ in which the variance of temperature $\overline{T'^2}$ is present:

$$G_{U_i'T'} = \beta g_i \overline{T'^2}\ . \qquad (12)$$

For a detailed description of the buoyancy, a transport equation for $\overline{T'^2}$ is solved:

$$\frac{\partial \overline{T'^2}}{\partial t} + \overline{U_i'}\frac{\partial \overline{T'^2}}{\partial x_i} = \frac{\partial}{\partial x_i}\left(c_{TT}\frac{k^2}{\varepsilon} + \Gamma\right)\frac{\partial \overline{T'^2}}{\partial x_i} -$$

$$2\overline{U_i'T'}\frac{\partial \overline{T}}{\partial x_i} - 2\varepsilon_{T'} - 2\Gamma\left(\frac{\partial\sqrt{\overline{T'^2}}}{\partial x_i}\right)^2 . \qquad (13)$$

This equation contains the modelling of Spalding (1971) for the turbulent diffusive term and the modelling of Nagano and Kim (1988) to consider the low Peclet number effects. The dissipation rate of the variance of temperature $\varepsilon_{T'}$ can be modelled using the definition of the turbulent time-scale ratio R:

$$\varepsilon_{T'} = \frac{\varepsilon\overline{T'^2}}{2Rk}. \qquad (14)$$

In this modelling, R is assumed to be constant. This assumption is not satisfactory because R depends on the Reynolds number, on the form of the flow and on the molecular Prandtl number (see Wörner and Grötzbach 1994). For this reason, and because R has an influence through the buoyancy term $G_{U'T'}$ and the new modelling term Z_i on the calculation of the heat fluxes, a transport equation for $\varepsilon_{T'}$ is solved in TMBF:

$$\frac{\partial \varepsilon_{T'}}{\partial t} + \overline{U_i'}\frac{\partial \varepsilon_{T'}}{\partial x_i} = \frac{\partial}{\partial x_i}\left(c_{DD}\frac{k^2}{\varepsilon} + \Gamma\right)\frac{\partial \varepsilon_{T'}}{\partial x_i} -$$

$$\varepsilon_{T'}\left(c_{D1}\frac{\varepsilon_{T'}}{\overline{T'^2}} + c_{D2}\frac{\varepsilon}{k} + c_{P1}\frac{\overline{U_i'T'}}{\overline{T'^2}}\frac{\partial\overline{T}}{\partial x_i} - c_{P2}\frac{P_k}{k}\right) +$$

$$2\Gamma\Gamma_t\left(\frac{\partial^2\overline{T}}{\partial x_i \partial x_j}\right)^2 . \qquad (15)$$

The modelling proposals of Hanjalic (1994) are used for this equation.

TMBF contains seven transport equations and 18 empirical coefficients. The range of application of all transport equations covers both the high Peclet numbers and the low Peclet numbers. The standard set of empirical coefficients (Table 1) from Gibson and Launder (1978) and Nagano and Kim (1988) is used in TMBF with the exception of the new empirical coefficient c_{T5} which was determined by Carteciano (1996).

Table 1: Standard set of empirical coefficients in TMBF.

k-tr.eq.		**ε-tr. eq**		$\overline{U_i'T'}$ **-tr. eq**.	
coeff.	value	coeff.	value	coeff.	value
σ_k	1.0	σ_ε	1.3	c_{TD}	0.11
c_μ	0.09	$c_{\varepsilon 1}$	1.44	c_{T1}	3.0
		$c_{\varepsilon 2}$	1.92	c_{T2}	0.33
		$c_{\varepsilon 3}$	0.8	c_{T3}	0.5
				c_{T4}	0.5
				c_{T5}	0.001
$\overline{T'^2}$ **-tr.eq.**		$\varepsilon_{T'}$**-tr.eq**.			
coeff.	value	coeff.	value	coeff.	value
c_{TT}	0.13	c_{DD}	0.13	c_{P1}	1.8
		c_{D1}	2.2	c_{P2}	0.72
		c_{D2}	0,8		

3. THE k-ε-σ_t MODEL

The common k-ε-σ_t model is a first-order 2-equation model. It consists of the transport equations of k (eq. 4) and ε (eq. 5) using an isotropic eddy viscosity (eq. 2) and of a simple isotropic heat flux approximation with a constant turbulent Prandtl number:

$$\Gamma_t = \frac{\nu_t}{\sigma_t}. \qquad (16)$$

This means, the Reynolds analogy is assumed. The turbulent heat fluxes are calculated using the gradient assumption of Fourier:

$$-\overline{U_i'T'} = \Gamma_t\frac{\partial\overline{T}}{\partial x_i}. \qquad (17)$$

The buoyancy term G_k is modelled using eq. 16 and eq. 17:

$$G_k = \beta g_i \frac{\nu_t}{\sigma_t} \frac{\partial \overline{T}}{\partial x_i}. \qquad (18)$$

For the turbulent Prandtl number the standard values of 0,9 for wall flows and 0,6 for free flows are used here in the calculations.

4. VALIDATION WITH FORCED CONVECTION

4.1 Experiments

The validation of TMBF with forced convection was carried out simulating numerically the following 2-D turbulent flows at high Reynolds numbers, Fig. 1: 1) a turbulent flow in a water channel downstream of a multibore jet block ejecting heated water from the central bore with the fluid leaving the bores at identical velocities (Krebs 1979); 2) a heated axisymmetric turbulent free jet of sodium (Knebel et al. 1993); 3) a heated wake flow of air behind a horizontal splitter-plate (Tureaud et al. 1988).

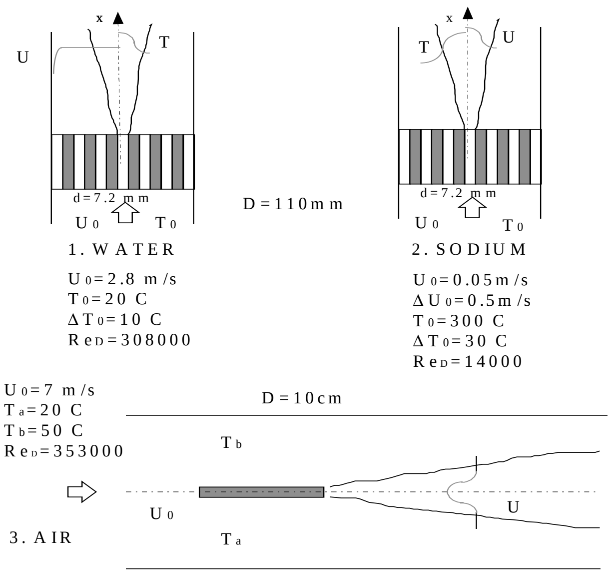

Figure 1: Sketch of 1) a thermal jet in water behind a multibore jet block; 2) a free jet in sodium behind a multibore jet block and 3) a heated wake flow of air behind a horizontal splitter plate.

The air flow above the flat plate was kept at a uniform temperature, which was different from that of the flow below the plate. The case of a stable temperature stratification of about $\Delta T=30^0$C is examined here.

These kinds of flows were primarily selected for two reasons: the Reynolds analogy is not valid, and a large range of the Prandtl number can be examined. Due to the first reason, separate treatment of the turbulent transport of heat and momentum is required for the turbulence model.

The description of the case specifications regarding the discretization parameters of the grids and the boundary conditions is given in Carteciano (1996).

4.2 Numerical Results

The results for forced convection show that TMBF can reproduce the mean temperature field well by using the standard values of empirical coefficients in all experiments, as illustrated in Figs. 2-4. This good agreement is due to the separate treatment in TMBF of the turbulent transport of heat and momentum. In contrast, the k-ε-σ_t model, which assumes the Reynolds analogy, cannot accurately simulate the mean temperature field when the standard turbulent Prandtl numbers are used. As shown in Figs. 5-7, the k-ε-σ_t model overestimates with eq. 17 the turbulent heat flux perpendicular to the flow direction because of the very high temperature gradients in this direction. This turbulent heat flux determines the mean temperature field. The k-ε-σ_t model can give a good simulation only by adjusting the value of σ_t (see Carteciano 1995 and 1996).

TMBF simulates properly the variance of temperature $\overline{T'^2}$ in all experiments (Figs. 8-10). Agreement is achieved by using the transport equation of $\varepsilon_{T'}$. The different values calculated for the turbulent time-scale ratio R in each experiment (Carteciano 1996) confirm the dependence of R on the molecular Prandtl number, on the Reynolds number and on the origin of turbulence.

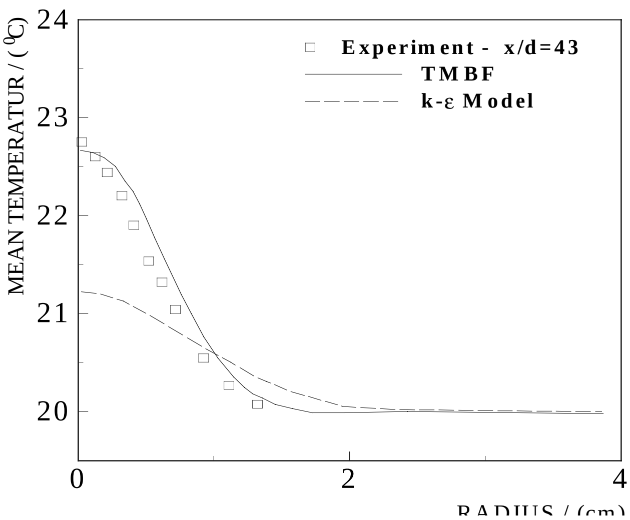

Figure 2: Radial profile of mean temperature. Thermal jet in water.

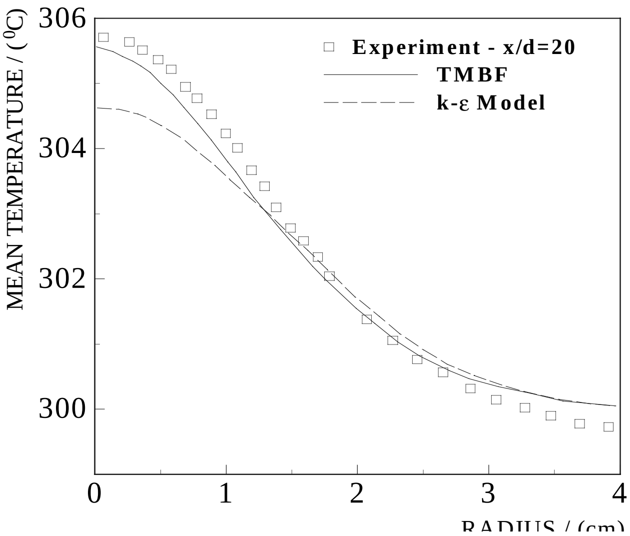

Figure 3: Radial profile of mean temperature. Free jet in sodium.

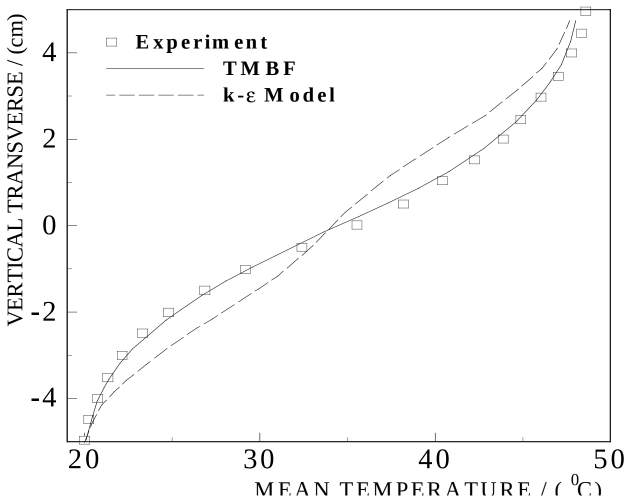

Figure 4: Transverse profile of mean temperature. Wake flow in air.

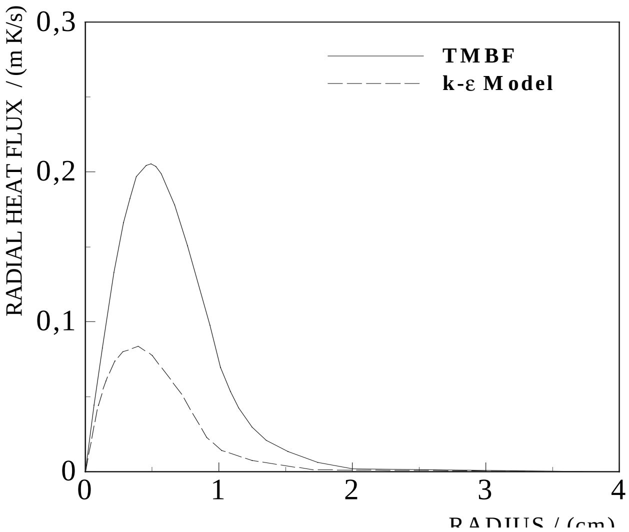

Figure 5: Radial profile of radial turbulent heat flux. Thermal jet in water

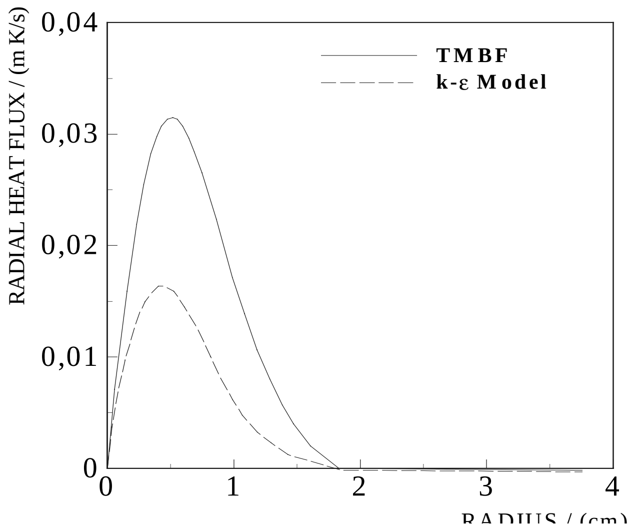

Figure 6: Radial profile of radial turbulent heat flux. Free jet in sodium.

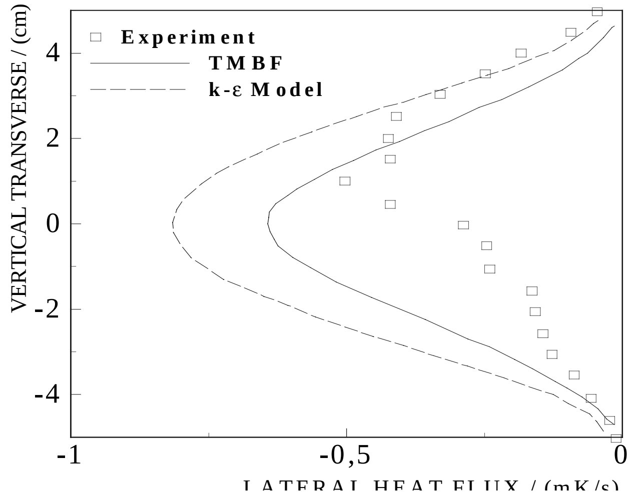

Figure 7: Transverse profile of lateral turbulent heat flux. Wake flow in air.

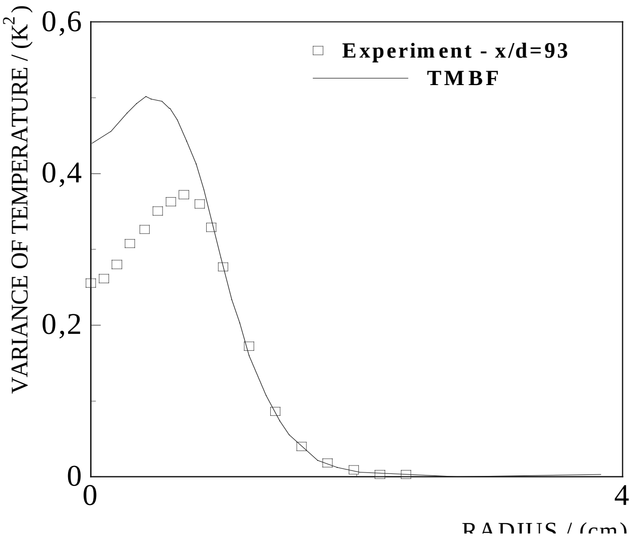

Figure 8: Radial profile of variance of temperature. Thermal jet in water.

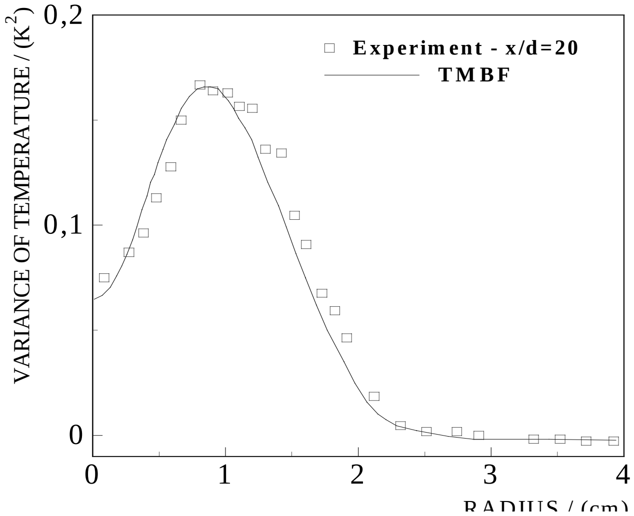

Figure 9: Radial profile of variance of temperature. Free jet in sodium.

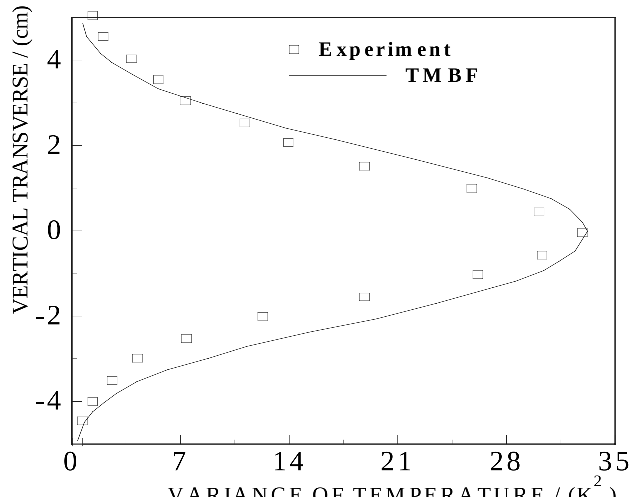

Figure 10: Transverse profile of variance of temperature. Wake flow in air.

5 VALIDATION WITH MIXED CONVECTION

5.1 Experiment

TMBF was validated for mixed convection by means of experimental data obtained for 2-D axisymmetric wake flow behind a heated sphere in a vertical water channel (Fig. 11, Suckow 1993). The mean velocity field of the upward flow behind the sphere is the result of non-linear interaction of typical wake and buoyancy effects. The radial turbulent heat flux is responsible for the radial spreading of the vertical thermal jet behind the sphere, whereas a strong vertical component of heat flux is produced by buoyancy. For this reason, both components of heat flux must be accurately simulated by a turbulence model.

The boundary conditions and the discretization parameters of the grids for the numerical simulation are specified in Carteciano (1996).

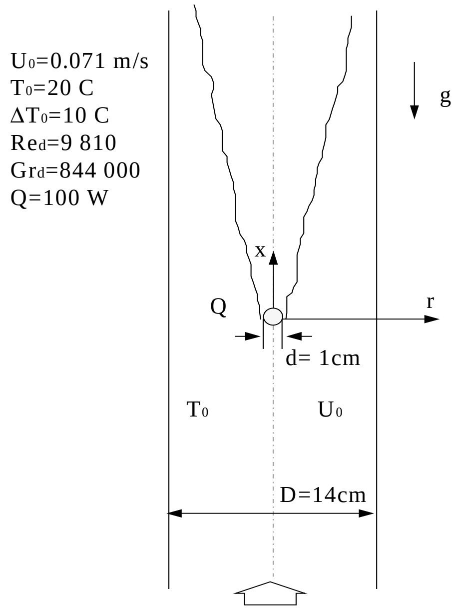

Figure 11: Sketch of a wake flow behind a heated sphere in a vertical water channel.

5.2 Numerical Results

TMBF reproduces well the radial spreading of the thermal jet by including the new modelling Z_i in the transport equation of the heat fluxes (Fig. 12). For the same reason as in the case of forced convection, the radial spreading of the temperature jet is overestimated by the k-ε-σ_t model using the standard value of the turbulent Prandtl number σ_t.

Figure 13 shows a typical radial profile of the mean velocity in the axial direction. The calculation with TMBF predicts the wake flow well, with an increase in the mean velocity in the vicinity of the axis, which is produced by buoyancy. The calculation using the k-ε-σ_t model does not show this increase; no buoyancy is calculated by the k-ε-σ_t model. The improved prediction of TMBF is primarily due to its detailed description of the buoyancy. Near the axis, the buoyancy forces produce an increase of the variance of temperature $\overline{T'^2}$ and, in particular, of the turbulent heat flux in the axial direction. These turbulent quantities, which are described in TMBF by means of transport

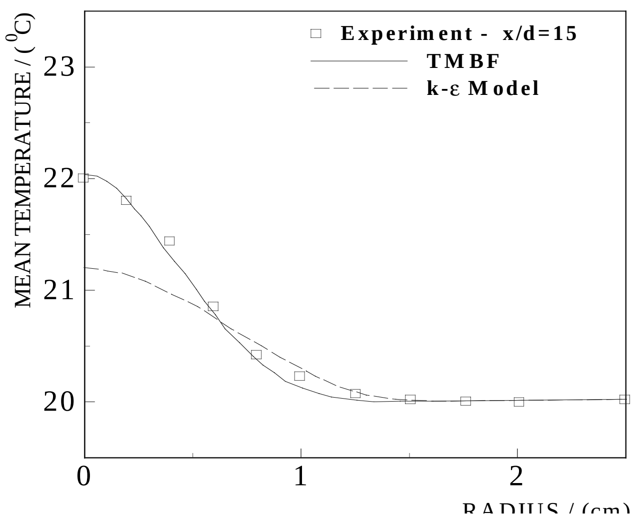

Figure 12: Radial profile of temperature. Wake flow behind a heated sphere in water.

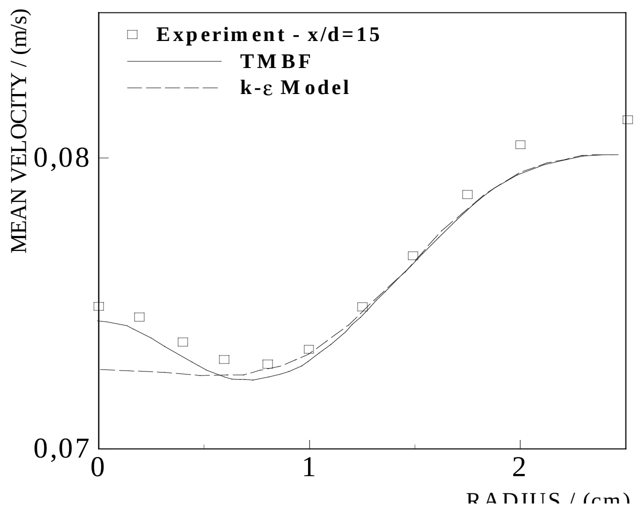

Figure 13: Radial profile of velocity. Wake flow behind a heated sphere in water.

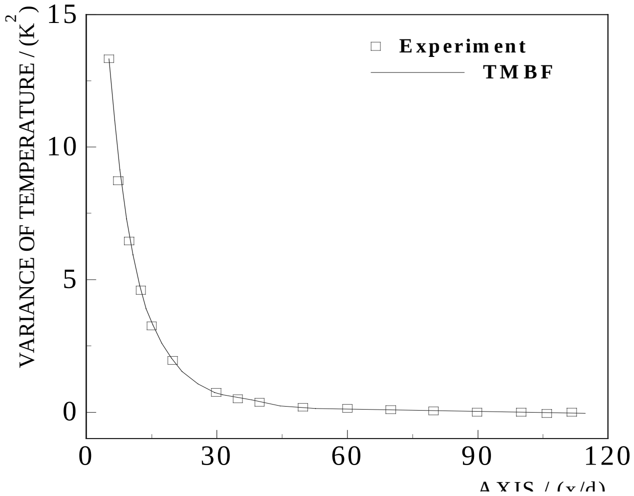

Figure 14: Axial profile of variance of temperature. Wake flow behind a heated sphere in water.

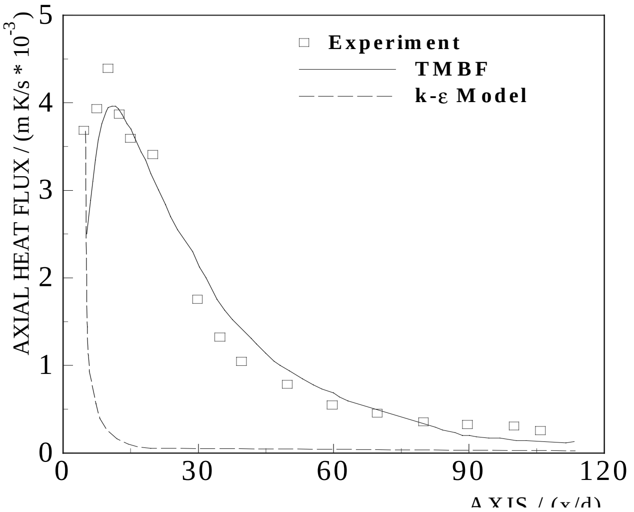

Figure 15: Axial profile of axial turbulent heat flux. Wake flow behind a heated sphere in water.

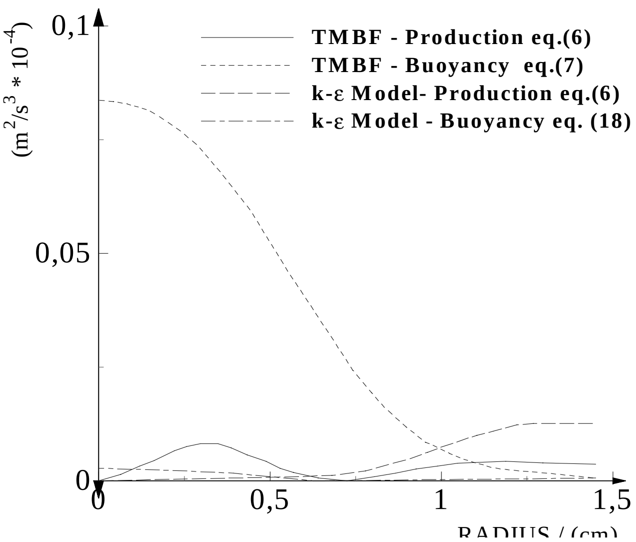

Figure 16: Radial profile of production and buoyancy terms in the transport equation of k. Wake flow behind a heated sphere in water.

equations and connected by the buoyancy term $G_{U'T'}$, are well predicted by TMBF, as shown by the axial profiles in Figs. 14 and 15. Due to the good prediction of the axial turbulent heat flux, the buoyancy term G_k (eq. 7) becomes the main production term in the transport equation of k (Fig. 16). This term is responsible, through the high production of k near the axis, for the increase of the mean velocity in the vicinity of the axis. In contrast, the turbulent heat flux in the axial direction with the k-ε-σ_t model, which is calculated with eq. 17, becomes very small because of the small axial gradient of mean temperature (Fig. 15). The buoyancy term G_k becomes negligible in the transport equation of k (Fig. 16). Therefore, this description fails to predict adequately the buoyancy in this experiment

6. CONCLUSION

A turbulence model for buoyant flows (TMBF) was developed as a combination of a first-order 2-equation model for the turbulent transport of momentum and of a second-order 5-equation model for the turbulent transport of heat. TMBF has been implemented in the computer code FLUTAN and tested for the following kinds of flows: 2-D forced convection flows with different fluids, where the assumptions of isotropy of the eddy heat diffusivity and of Reynolds analogy are not valid; 2-D buoyant flow with mixed convection. Calculations were compared with experimental data and with calculations using the k-ε-σ_t model.

The results show that TMBF can adequately simulate the field of mean temperature using the standard values of empirical coefficients in all experiments. For buoyant flow with mixed convection, new modelling in the transport equations for heat fluxes has been validated.

TMBF can also simulate well the influence of the mean temperature field on the mean velocity field by buoyancy in the flow with mixed convection. This good agreement is due to the detailed description of buoyancy in TMBF. All turbulent quantities which are important for the buoyancy, such as variance of temperature and turbulent heat fluxes, are described by transport equations.

The k-ε-σ_t model cannot accurately simulate the mean temperature field in these experiments and fails to predict the buoyancy in the mixed convection flow. This is due to the assumptions of Reynolds analogy and of isotropy of the eddy heat diffusivity. In 2-D forced convection flows, only one component of the turbulent heat fluxes is important for the simulation of the field of mean temperature. The assumption of isotropy can be overcome here because one must well simulate only this component. This can be achieved in the k-ε-σ_t model by adjusting the value of the turbulent Prandtl number. In contrast, two components of the heat fluxes determine, in buoyant flow with mixed convection, the mean temperature field and the influence on the velocity field of buoyancy. This means that both components must be well simulated. Therefore, the incorrect assumption of isotropy of eddy heat diffusivity cannot be overcome in the k-ε-σ_t model just by adjusting the value of σ_t. An anisotropic or even a second-order description, as in TMBF, is necessary for this.

The field of the variance of temperature was well simulated by TMBF in all experiments, due to the use of a transport equation for $\varepsilon_{T'}$. This allows a non-constant value of R to be calculated by eq.14, which is also important for the new modelling term Z_i (eq. 11).

TMBF has been successfully validated in forced and mixed convection flows for two-dimensional cases and it is clearly an improvement on the k-ε-σ_t model, above all, in mixed convection flow. Further validation of TMBF should be carried out for flows with natural convection, but only in two-dimensional cases. In fact, the range of validity of TMBF could be restricted by the assumption of isotropy of the eddy viscosity ν_t. This incorrect assumption is not significant in 2-D flows because only one component of the Reynolds stresses is important for the field of mean velocity. This is not the case with 3-D flows. For these kinds of flows, ASM extension or transport equations for the Reynolds stresses must be used.

NOMENCLATURE

g	m/s^2	gravitational acceleration
i,j,k	-	radial, azimuthal and axial direction
k	m^2/s^2	turbulent kinetic energy
n	-	normal direction to a wall
R	-	turbulent time-scale ratio
Re	-	Reynolds number
$Re_\tau=U_\tau y/\nu$	-	local Reynolds number
$Re_t=k^2/\nu\varepsilon$	-	turb. Reynolds number
Ri_f	-	flux Richardson number
p	N/m^2	pressure
Pe=Re Pr	-	Peclet number
$Pe_t=Re_t$ Pr	-	turb. Peclet number
Pr	-	Prandtl number
$\overline{T}$	K	mean temperature
$\overline{T'^2}$	K^2	variance of temperature
t	s	time
$\overline{U}$	m/s	mean velocity
$U_\tau=(\tau_w/\rho)^{1/2}$	m/s	friction velocity
$\rho\,\overline{UiUj}$	kg/m s^2	Reynolds stresses
$\rho\,\overline{U_i'T'}$	kg K/m^2 s	turbulent heat fluxes
x	m	space co-ordinate
y	m	normal distance to the axis
β	1/K	volumetric expansion coefficient
Γ	m^2/s	molecular heat diffusivity
Γ_t	m^2/s	eddy heat diffusivity
δ_{ij}	-	Kronecker delta
ε	m^2/s^3	dissipation rate of k
$\varepsilon_{T'}$	K^2 /s	dissipation rate of $\overline{T'^2}$
ρ	kg/m	density
ν	m^2/s	molecular viscosity
ν_t	m^2/s	eddy viscosity
σ_t	-	turbulent Prandtl number
τ_w	N/m^2	wall shear stress

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