Turbulent Heat Transfer in an Internally Heated Fluid Layer

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SUMMARY

The method of direct numerical simulation is used to investigate turbulent convection in an internally heated fluid. Two simulations of channels with very different horizontal extensions and different grid widths are analysed. The large scale structures of the flow are completely recorded in the larger channel. The calculated statistical turbulence data of both cases are comparable and even for the smaller channel quite accurate. The reason is found in analyzing a computer generated movie: The macroscopic length scale is kept small by the Rayleigh-Taylor instability.

1. Introduction

The convection in horizontal fluid layers with homogeneous internal heating is a model for certain astrophysical, geophysical, environmental, and technical problems. Similar types of flow occur in the outer layer of some stars, in the earth mantle, in the bioconvection of swimming organisms, in chemical reactors, and in molten fuel of nuclear reactors. In the latter problem the heat transfer was investigated in detail, see [6], the turbulence, however, is investigated rarely, see e.g. the experiments with a heated lower wall [5]. For a channel with quasi infinite horizontal extension and isothermal walls turbulence data have been deduced from direct numerical simulations [2]. The simulations resolved all relevant small scale structures [1], but it remained an open problem whether the horizontal length of the channel was large enough to reproduce adequately the macroscopic scales of the flow.

To solve this problem an additional simulation is presented for a considerably larger channel than in [2]. By comparing both simulations the influence of the horizontal channel width is investigated on some statistical data of turbulence and on the flow structure.

2. Simulation model

The simulation model is the TURBIT-3 code [3]. It is based on the complete three-dimensional time-dependent conservation equations for mass, momentum, and thermal energy. The validity of the Boussinesq approximation is assumed. A finite difference scheme for these equations is deduced.
by applying Schumann’s method [7] to the derivatives in space. This provides equations for surface averaged velocities \( \bar{u} \) and volume averaged pressure \( \bar{p} \) and temperature \( \bar{T} \). Integration in time is performed by an explicit Euler-leap frog scheme. The resulting finite difference scheme is discretized on a staggered grid to get exact mass conservation. Subgrid scale terms are neglected as the grids will be chosen to record all relevant flow structures.

For normalization we use the channel height \( D \), time \( \Delta t_0 \), velocity \( u_0 = (g\Delta T_0 D)^{1/2} \), and temperature \( \Delta T_0 = \langle T_{\text{max}} - T_w \rangle \). The latter is calculated using the relation between Damköhler and Nusselt numbers at the lower and upper wall. \( Nu_1 \) and \( Nu_2 \); \( Da_0 = QD^2/\lambda \Delta T_0 = Nu_1 + Nu_2 \), with \( Q \) = volumetric heat source and \( \lambda \) = thermal conductivity. The Prandtl number is \( Pr = v/\alpha \) with \( v \) = viscous diffusivity and \( \alpha \) = thermal diffusivity, the Rayleigh number is \( Ra = g \beta Q D^5/(v \alpha) \) with \( g \) = gravity and \( \beta \) = volume expansion coefficient.

3. Case specifications

The convection of an internally heated fluid bounded by two horizontal rigid walls is considered. In both horizontal directions periodicity is assumed with periodicity lengths \( X_i \), Table 1. In the new simulation, case 12, \( X_i \) is more than twice the value as used in case 11 taken from [1,2,3] and more than four times the wavelength of the flow pattern observed at onset of convection. As the wavelength is expected to decrease with increasing \( Ra \) at this type of flow, case 12 should show no deficiencies due to too short a periodicity length. In the vertical direction the velocities are set to zero at the walls and the wall shear stresses and wall heat fluxes are approximated by linear finite difference approximations. The wall temperatures \( T_w \) are set equal and kept constant in space and time.

Table 1: Case specifications and time intervals (i = 1, 2).

<table>
<thead>
<tr>
<th>Case</th>
<th>( N_i )</th>
<th>( N_3 )</th>
<th>( X_i )</th>
<th>( \Delta x_i )</th>
<th>( \Delta x_{3w2} )</th>
<th>( N_t )</th>
<th>( t_{\text{max}} )</th>
<th>( t_{\text{CPU}} )</th>
<th>( N_{tav} )</th>
<th>( \Delta t_{tav} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>64</td>
<td>32</td>
<td>2.8</td>
<td>0.04375</td>
<td>0.020</td>
<td>4,040</td>
<td>40.2</td>
<td>1.5 h</td>
<td>15</td>
<td>5.6</td>
</tr>
<tr>
<td>12</td>
<td>180</td>
<td>32</td>
<td>7.2</td>
<td>0.04</td>
<td>0.005</td>
<td>101,920</td>
<td>88.7</td>
<td>223.1 h</td>
<td>17</td>
<td>20.8</td>
</tr>
</tbody>
</table>

The simulations are started from zero velocity and from approximated vertical temperature profiles on which random fluctuations are superposed. The initial \( \Delta T_0 \) is calculated by means of correlations given in [4] which results in \( Da_0 = 15.7 \). The simulations are stopped after \( N_t \) time steps at time \( t_{\text{max}} \) when the Nusselt numbers at both walls (Fig. 1) and the equivalent value of the turbulence energy in the channel analysed from the time-dependent results oscillate around mean values, this means when fully developed flow is simulated for a certain time. The computing times \( t_{\text{CPU}} \) are given in equivalent hours for the current code version with improved vectorization and a SIEMENS/FUJITSU VP50 vector computer with a storage of 64 MBytes, Tab. 1.

4. Simulation Results

The complete three-dimensional results for the three velocity components, pressure, and temperature are stored for a certain number of time steps. To obtain reasonable statistical data from the time dependent results, averages \( \langle y \rangle \) are formed for a calculated variable \( y \) over horizontal planes, and these are averaged over \( N_{tav} \) time steps distributed within the period \( \Delta t_{tav} \) at the end of the simulated problem time, see Table 1. The tremendous increase in \( \Delta t_{tav} \) from case 11 to 12 reduces the statistical coupling in time, compare Fig. 1, and that of \( X_i \) the coupling in space.

4.1 Verification

The Nusselt numbers calculated at both walls for case 11 are in the middle of the scatter band of experimental data [1,6], and those calculated for case 12 agree as well. Fig. 1 shows that the Nusselt numbers, determined by horizontal averaging only, achieve deviations from the mean value below \( \pm 2.5\% \) even after less than half the problem time.

The calculated vertical time mean temperature profiles are compared to the experimental data from Kulacki and to the two-dimensional calculation by Emara [6] in Fig. 2. Both simulations differ by 3.5% from one another. The reason might be that the time \( t_{\text{max}} \) simulated with the coarser grid is too small to really get fully developed flow, see Table 1 and Fig. 1. In the boundary layers both simulations agree as well with the experimental data.
Fig. 3: Vertical cuts of calculated temperature fields at any time and interferograms from experiments by Jahn. The contour line increment is $\Delta = 0.08$ for case 11 and 0.125 for case 12.

as the Nusselt numbers do. In the region $0.3 < x_1 < 0.8$ a somewhat larger increase in temperature is predicted by both models than in the experiment.

The local convection phenomena are compared to experiments by cuts through the temperature field. In the vertical cuts one finds the internally heated hotter fluid bounded by the colder walls, Fig. 3. The lower boundary layer is stably stratified. The core is weakly stable. The upper boundary layer is unstable and therefore drives the flow. Here cold plumes develop, fall, and some of them deeply penetrate into the lower layer. Both simulations reproduce the structures of the plumes as observed in the experiments [4].

All three results give about the same vertical temperature distribution. Thus, the predicted temperature increase in Fig. 2, is verified by this experiment.

Horizontal cuts are given for a plane near the temperature maximum $T_{\text{max}}$ in Fig. 4. Most of the cold plumes form cells around a hot isothermal core. The cells are connected randomly. Some smaller plumes occur in larger cells. Again, the structures simulated are comparable to and agree with those in the experiment [4].

So far the simulation results are compared to all types of directly recorded experimental data available for this flow at the parameters considered, and no problem with the smaller periodicity length becomes obvious.

4.2 Predicted turbulence data

In this chapter the influence of the periodicity lengths on the predicted turbulence data is investigated.

The root-mean-square (rms) values for the velocity fluctuations show that in the inner part of the channel most energy is in the vertical component, Fig. 5. The horizontal components are more important only within the boundary layers. The data for both horizontal components of case 12 roughly coincide whereas the result for the smaller channel shows statistical errors due to insufficient averaging. This deviation is completely associated with the component $i = 2$ in case 11; all other curves coincide nearly totally.

The pressure strain terms, which are also called the tendency-toward-isotropy terms, show near midplane minor energy transfer from the vertical to the horizontal velocity fluctuations, Fig. 6. This indicates that the down-
Fig. 6: Calculated pressure-strain terms

Fig. 7: Calculated rms-values of temperature fluctuations

Fig. 8: Calculated vertical turbulent heat flux

The turbulent heat flux in the vertical direction is calculated to be negative in the lower, mainly conduction controlled, boundary layer, Fig. 8. In the upper three quarters of the channel the flux is directed upwards. As the temperature maximum is at $x_3 = 0.77$ we find an extensive region with countergradient turbulent heat flux. This phenomenon is discussed in [2, 3], and reasons are given in [8]. Both simulations agree except at the maximum. Since the vertical velocity and temperature fluctuations of both cases do not differ the difference in the turbulent heat flux can only be due to a not fully developed flow or insufficient averaging of case 11.

4.3 Macroscopic length scale

Two-point correlations $R_{ij}(x_1) = \langle u_i'(x) u_j'(x + x_1) \rangle / \langle u_i'(x) u_j'(x) \rangle$ are calculated from the velocity fluctuations slightly above midplane, Fig. 9. The rapid decrease at small displacements is identical in both simulations. At $x_1 > 0.3$ case 12 gives a somewhat flatter decrease for $R_{22}$ than case 11. The zero crossings for $u_1$ and $u_3$ in both cases occur at about the same position; this means the typical wavelength $\lambda$ of the large scale structures is roughly the same in both simulations. With $\lambda \approx 0.6$ it is much smaller than $\lambda_c = n/2$ at onset of convection. The strong oscillations and non-zero values at $x_1 > 0.8$ in case 11 show that the smaller periodicity length causes statistical spatial coupling which cannot be detected in Fig. 4. The other simulation is clearly free of this problem.

Fig. 9: Two-point velocity auto-correlations in $x_1$ direction at $x_3 = 0.504$ (case 11) and $x_3 = 0.554$ (case 12).

Fig. 10: Time-dependent horizontal quarter sections through the temperature field; case 12, $\Delta = 0.0625$, $x_3 = 0.953$. 
The meaning of the length scale deduced from $R_i$ is investigated on the basis of a movie which presents calculated time-dependent flow structures within the upper boundary layer, Fig. 10. Some smaller open or closed cells contract to knots and widen the neighboring cells. Mainly in the larger cells round plumes develop, grow to plane plumes, merge with other ones, and form new cells. The range of wavelengths $\lambda_{max}$ covered by the cells is between zero and about 1.7. The plumes developing within cells reduce the wavelength to 0.5 to 0.8. This is in accordance with the value estimated from $R_i$ and also with $\lambda = 0.7$ determined from the energy spectra for case 11 [3]. As in fact in all larger cells plumes develop which reduce the length-scale one might deduce the wavelength found is about the critical one for the Rayleigh-Taylor instability of the upper boundary layer.

5. Conclusions

Both simulations with differing small scale resolution and very different horizontal extension could be verified by all types of experimental data published for this type of flow. The results confirm that the requirements [1] to resolve the relevant small scales are sufficient for calculation of the turbulence data considered here. In these data both simulations differ only slightly due to insufficient averaging in the analysis of the older simulation with the smaller periodicity length. Nevertheless, both simulations can be used as a data base to analyze any turbulence data, but the new one will be more suited for data mainly determined by large scale structures.

The new simulation also confirms that the macroscopic length scale of this flow decreases with increasing Rayleigh number. The movie shows that the Rayleigh-Taylor instability keeps the always growing length scale small. From this and from the analyzed two-point correlations it follows that in future simulations the periodic length needs not to exceed the $2 \lambda_{max}$ observed visually or the 4 to 5 $\lambda$ calculated. This allows to perform accurate studies for similar Rayleigh numbers by means of direct numerical simulations with grids somewhat larger than the coarser one used here. With the node numbers of the larger grid highly turbulent flows can be investigated in present day computer systems up to about $10^4$ times the critical Rayleigh number. Thus, the method of direct numerical simulation may be used for parameter ranges which were only covered by experiments, if any.

6. Acknowledgements

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7. References

Refined Flow Modelling and Turbulence Measurements


The utilization of high-speed, large-capacity computers has contributed much to the development of accuracy in numerical models of fluid phenomena in the fields of nuclear power, energy, environment, construction, geophysics, etc. On the other hand, highly advanced modeling of flow is now possible thanks to greater progress in the measurement of few, and particularly in the techniques of turbulence measurement. Specifically, the experimental data to validate computations are very important as input to the numerical modeling, and they are of deep significance to discussions of effects which are not yet covered by numerical models. The objective of the Symposium is to bring together scientists and engineers, working in the fields mentioned above, to discuss the latest advances in flow modeling and in turbulence measurements.

The lectures and papers selected have been re-edited into twelve parts including keynote lectures delivered by Professors W. Rodi, W. Z. Sadeh and M. Akiyama, which highlighted the state-of-the-art in the computational fluid dynamics, experimental fluid dynamics and applications.

Topics of papers gathered in this book range from modeling in turbulent flows and heat transfer, progress in numerical techniques, dispersion, stratified flow in atmosphere and water environments to flows in rivers and open channels.